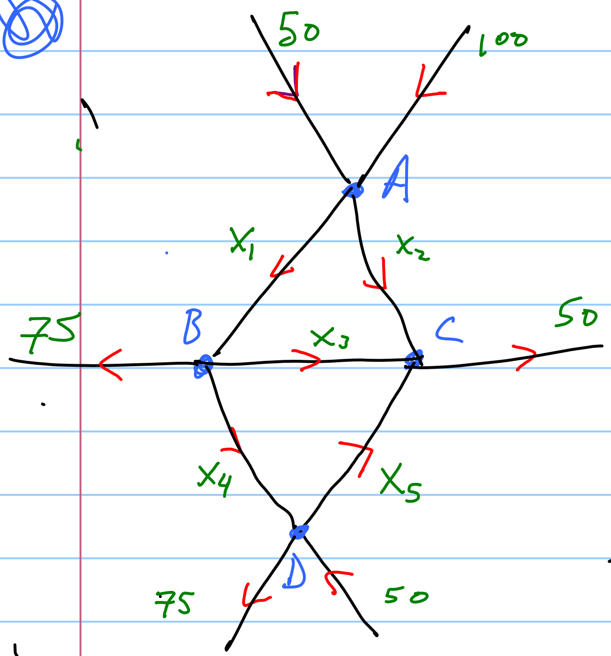
Applications of Gauss Elimination



Traffic In = Traffic out

A : $100 + 50 = x_1 + x_2$

B : $x_1 = 75 + x_3 + x_4$

C : $x_2 + x_3 + x_5 = 50$

D : $x_4 + 50 = 75 + x_5$

$$\begin{cases} x_1 + x_2 = 150 \\ x_1 - x_3 - x_4 = 75 \\ x_2 + x_3 + x_5 = 50 \\ x_4 - x_5 = 25 \end{cases}$$

$$\begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \end{matrix} \\ \left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 150 \\ 1 & 0 & -1 & -1 & 0 & 75 \\ 0 & 1 & 1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 & -1 & 25 \end{array}\right) \end{array} \quad R_2 - R_1 \to R_2$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 150 \\ 0 & -1 & -1 & -1 & 0 & -75 \\ 0 & 1 & 1 & 0 & 1 & 50 \\ 0 & 0 & 0 & 1 & -1 & 25 \end{array}\right) \quad R_3 + R_2 \to R_3$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 150 \\ 0 & -1 & -1 & -1 & 0 & -75 \\ 0 & 0 & 0 & -1 & 1 & -25 \\ 0 & 0 & 0 & 1 & -1 & 25 \end{array}\right) \quad R_4 + R_3 \to R_4$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 150 \\ 0 & -1 & -1 & -1 & 0 & -75 \\ 0 & 0 & 0 & -1 & 1 & -25 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$-R_2 \to R_2$

$-R_3 \to R_3$

$$\begin{array}{c} \left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 150 \\ 0 & 1 & 1 & 1 & 0 & 75 \\ 0 & 0 & 0 & 1 & -1 & 25 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \\ \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 & \end{matrix} \end{array}$$

Backsolve Let $x_5 = S_1$

Row 3 $\begin{cases} x_4 - S_1 = 25 \\ x_4 = 25 + S_1 \end{cases}$

Let $x_3 = S_2$

Row 2 $\left|\begin{array}{l} x_2 + s_2 + 25 + s_1 = 75 \\ x_2 = 50 - s_2 - s_1 \end{array}\right.$

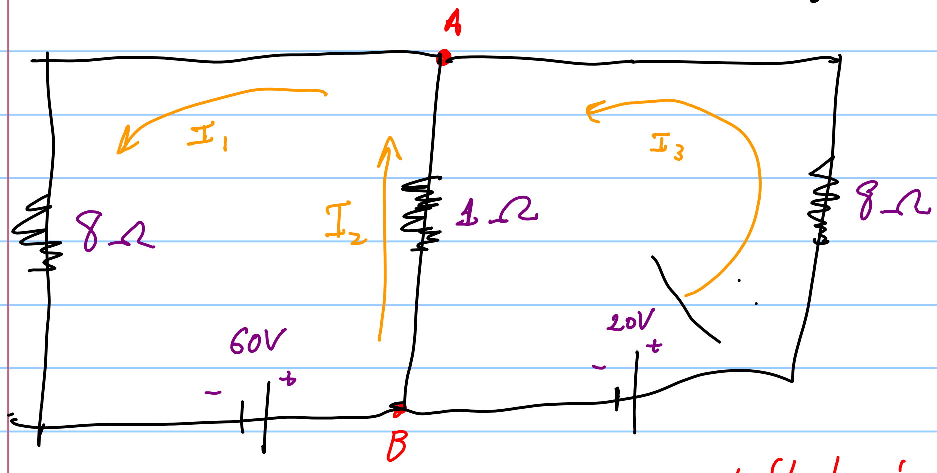
Row 1 $\left|\begin{array}{l} x_1 + 50 - s_2 - s_1 = 150 \\ x_1 = 100 + s_2 + s_1 \end{array}\right.$

<u>General Solution</u> $(x_1, x_2, x_3, x_4, x_5) = (100 + s_2 + s_1,\ 50 - s_2 - s_1,\ s_2,\ 25 + s_1,\ s_1)$

<u>Domain Restrictions</u> $s_1 \geq 0$, $s_2 \geq 0$, $s_1 + s_2 \leq 50$

Current Flow Diagrams

<u>Rules:</u>
1. Traffic Flow Rules
2. The sum of the voltage drops around any loop must be 0
3. The voltage drop E across a resistor with resistance R and current I passing through is given by $E = IR$



Resistor

Power Source

A: $I_2 + I_3 = I_1$

B: $I_1 = I_2 + I_3$] Ignore

Left loop: $1 \cdot I_2 + 8I_1 - 60 = 0$

Right loop: $8I_3 - 1 \cdot I_2 - 20 = 0$

[Big loop: $8I_1 + 8I_3 - 80 = 0$

$$\begin{aligned} I_1 - I_2 - I_3 &= 0 \\ 8I_1 + I_2 \quad\quad &= 60 \\ -I_2 + 8I_3 &= 20 \end{aligned}$$

$$\begin{array}{c} \begin{array}{ccc} I_1 & I_2 & I_3 \end{array} \\ \left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 8 & 1 & 0 & 60 \\ 0 & -1 & 8 & 20 \end{array}\right) \end{array} \quad R_2 - 8R_1 \to R_2$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 9 & 8 & 60 \\ 0 & -1 & 8 & 20 \end{array}\right) \quad R_2 + 9R_3 \to R_2$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 0 & 80 & 240 \\ 0 & -1 & 8 & 20 \end{array}\right) \begin{array}{l} \\ \frac{1}{80}R_2 \to R_2 \\ -R_3 \to R_3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & -8 & -20 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

Chemical Equations

$$PbO_2 + HCl \longrightarrow PbCl_2 + Cl_2 + H_2O$$

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

Pb : $x_1 = x_3$

O : $2x_1 = x_5$

H : $x_2 = 2x_5$

Cl : $x_2 = 2x_3 + 2x_4$

$$\begin{cases} x_1 - x_3 = 0 \\ 2x_1 - x_5 = 0 \\ x_2 - 2x_5 = 0 \\ x_2 - 2x_3 - 2x_4 = 0 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & -2 & -2 & 0 & 0 \end{array}\right) \quad R_2 - 2R_1 \to R_2$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 1 & -2 & -2 & 0 & 0 \end{array}\right) \quad R_4 - R_3 \to R_4$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & -2 & 2 & 0 \end{array}\right) \quad R_2 + R_4 \to R_2$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & -2 & 2 & 0 \end{array}\right) \begin{array}{l} \\ \frac{1}{-2}R_2 \\ \\ -\frac{1}{2}R_4 \end{array}$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -2 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{2} & 0 \end{array}\right)$$

One free variable. This makes ¢ ....

Leontief Input-Output Economic Models

Suppose there are 3 industries A, B, C that all purchase from each other + themselves. (Ch 1 #16)

		proportion of output of industry		
		A	B	C
purchased by industry	A	0.35	0.50	0.30
	B	0.25	0.20	0.30
	C	0.40	0.30	0.40

Let x_1 be the amount that Industry A produces
" x_2 " B "
" x_3 " C "

Input = Output

A: $0.35x_1 + 0.5x_2 + 0.3x_3 = x_1$

B: $0.25x_1 + 0.2x_2 + 0.3x_3 = x_2$

C: $0.4x_1 + 0.3x_2 + 0.4x_3 = x_3$

etc...

Ch 2 Matrices

First, an $m \times n$ matrix is an array of numbers with m rows + n colums like so

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \vdots \\ a_{31} & & \ddots & \vdots \\ \vdots & & & \vdots \\ a_{m1} & \cdots & \cdots & a_{mn} \end{pmatrix}$$

It therefore has $m \cdot n$ entries

Adding matrices

If 2 matrices have the same # of rows & the same # of columns, then we add them by adding the entries.

Eg $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{pmatrix} = \begin{pmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{pmatrix}$

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 7 & 8 \\ 9 & 10 \end{pmatrix} =$ Undefined

Properties If A, B, C, O are all $m \times n$ matrices, where O is the all-zero matrix, then

1. $(A+B)+C = A+(B+C)$
2. $A+B = B+A$
3. $A+O = A$
4. There exists a unique matrix A' so that $A+A' = O$

Scalar Multiplication

If A is any matrix, and if c is any real number, where

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

$A = (a_{ij})$

then $cA = \begin{pmatrix} c\,a_{11} & \cdots & c\,a_{1n} \\ \vdots & \ddots & \vdots \\ c\,a_{m1} & \cdots & c\,a_{mn} \end{pmatrix}$

Ex

$$\pi \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} \pi & 2\pi & 3\pi \\ 4\pi & 5\pi & 6\pi \end{pmatrix}.$$

Properties If A, B, O are $m \times n$ matrices, $c, d \in \mathbb{R}$ where O is the all-zero matrix, then

1. $c(A+B) = cA + cB$
2. $(c+d)A = cA + dA$
3. $0A = \mathcal{O}$

   zero

4. $c(dA) = (cd)A$
5. $c\mathcal{O} = \mathcal{O}$

HW

Ch 1: 9, 11, 13, 18, 19e

Skip 2.11, 2.13